JADS expert program 2019

Frans de Ruiter, CQM

Assignment: Portfolio optimization

You are given the sample average returns and sample variances of $N = 100$ stocks[1]. Balancing the average return and risk, you create the portfolio according to the *mean-variance* portfolio optimization model:

$$Maximize \sum_{i=1}^{N} \overline{r_i} * x_i - \alpha \sum_{i=1}^{N} \sum_{j=1}^{N} s_{ij} * x_i * x_j$$

$$such\ that: \sum_{i=1}^{N} x_i = 1$$

$$x_i \geq 0, \ \ i = 1, \dots, N,$$

here, the decision variable x_i is the amount of asset i chosen, $\overline{r_i}$ the sample average return of asset i, s_{ij} the sample covariance between asset i and asset j for all $i = 1, \dots, N, j = 1, \dots, N$. The parameter α is the so-called *risk-aversion* parameter.

Tasks

A. Implement and solve the mean-variance portfolio optimization model above in Python, using Pyomo, using risk-aversion parameter $\alpha = 0.012$. The sample means and sample covariances are given in the file "MeanVarPortfolio.xlsx". You can use the Jupyter notebook given that you can find at the bottom of the page www.fransderuiter.com/JADS.

B. Plot the means and variances of each asset. In the same figure, plot the (sample) mean and covariance of the optimal portfolio you found. Is this indeed a portfolio on the efficient frontier?

C. (**Bonus extension question**). In the current model *short-selling* is not allowed: you cannot have negative positions in assets ($x_i \geq 0$). Change the model formulation to allow for short positions. However, for this assignment a restriction put forth by the regulators is that the total short position cannot be more than 10% of the total long position.

   (Hint: Introduce two new variables $x_i^{long} \geq 0$ and $x_i^{short} \geq 0$, together with the constraint $x_i = x_i^{long} - x_i^{short}$. You still have introduce a constraint on the total short position to comply with the regulations regarding the total short position $\sum_{i=1}^{N} x_i^{short}$ and the total long position $\sum_{i=1}^{N} x_i^{long}$.)

[1] Original data for this case came from the website of Kennet French (from the famous Fama-French Model): http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html