Business Analytics
JADS expert program 2018

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Case: Portfolio optimization

You are given the sample average returns and sample variances of $N = 100$ stocks[1]. Balancing the average return and risk, you create the portfolio according to the *mean-variance* portfolio optimization model:

$$Maximize \sum_{i=1}^{N} \overline{r_i} * x_i - \alpha \sum_{i=1}^{N} \sum_{j=1}^{N} s_{ij} * x_i * x_j$$

$$such\ that: \sum_{i=1}^{N} x_i = 1$$

$$x_i \geq 0, \quad i = 1, \dots, N,$$

here, the decision variable x_i is the amount of asset i chosen, $\overline{r_i}$ the sample average return of asset i, s_{ij} the sample covariance between asset i and asset j for all $i = 1, \dots, N, j = 1, \dots, N$. The parameter α is the so-called *risk-aversion* parameter.

Tasks

A. Implement and solve the mean-variance portfolio optimization model above in Excel (or any other package of your choice) using risk-aversion parameter $\alpha = 0.012$. The sample means and sample covariances are given in the file "MeanVarPortfolio.xlsx". See also Excel tips on the next page.

B. Plot the (sample) mean and covariance of the portfolio found in the Figure given in the second sheet of the Excel file.

C. (**Bonus extension question*).* In the current model *short-selling* is not allowed: you cannot have negative positions in assets ($x_i \geq 0$). Change the model formulation to allow for short positions. However, a restriction put forth by the regulators is that the total short position cannot be more than 10% of the total long position.

   (Hint: Introduce two new variables $x_i^{long} \geq 0$ and $x_i^{short} \geq 0$, together with the constraint $x_i = x_i^{long} - x_i^{short}$. You still have introduce a constraint on

[1] Original data for this case came from the website of Kennet French (from the famous Fama-French Model): http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

the total short position to comply with the regulations regarding the total short position $\sum_{i=1}^{N} x_i^{short}$ and the total long position $\sum_{i=1}^{N} x_i^{long}$.)

Excel tips

Use the native excel solver (as the standard installation of OpenSolver does not come with a nonlinear solver). You can enable this in Excel 2016 via File → Options → Add-ins→ Go (in manage Excel add-ins) → Click on checkbox for solver add-in. If you encounter any trouble with this, or cannot find it in your excel version, then we can assist.

The way to calculate the value of the objective term $\sum_{i=1}^{N} \sum_{j=1}^{N} s_{ij} * x_i * x_j$ in Excel is described below. Suppose there are three assets, $N = 3$ (just for simplicity in the explanation below), and that the values of s_{ij} for all i and j equal to the numbers in the red cells in the figure below. The values for x_i are given in green.

	A	B	C	D	E	F	G	H
1								
2		*s_ij*	*1*	*2*	*3*			x_i
3		*1*	1	2	3		*1*	10
4		*2*	4	5	6		*2*	20
5		*3*	7	8	9		*3*	30
6								
7								
8			Sum of s_ij x_i x_j					
9			22800	=SUMPRODUCT(H3:H5;MMULT(C3:E5;H3:H5))				
10								

Then the value of $\sum_{i=1}^{N} \sum_{j=1}^{N} s_{ij} * x_i * x_j$ can be computed by the formula:

=SUMPRODUCT(H3:H5;MMULT(C3:E5;H3:H5)).

(In more technical mathematical terms, this calculates $x'Sx$, where x is a vector and S a matrix.)