Model formulation for the facility location model

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Parameters

Let $i = 1, \dots, N$ be the index for the cities.

d_i : demand at city i in number of pallets
F_i : fixed costs for opening a distribution center (DC) at location i in euros
S_i : capacity at DC i (if opened) in number of pallets
c_{ij} : costs for transporting one pallet from city i to city j in euros

Variables

$$x_i : \quad = \begin{cases} 1 \text{ if a DC is opened in city } i \\ 0 \text{ otherwise} \end{cases}$$

y_{ij} : number of pallets transported from city i to city j

Model

$$\begin{aligned}
\min_{x,y} \quad & \sum_{i=1}^{N} F_i x_i + \sum_{i=1}^{N} \sum_{j=1}^{N} c_{ij} y_{ij} \\
\text{s.t.} \quad & \sum_{i=1}^{N} y_{ij} = d_j \qquad j = 1, \dots, N \\
& \sum_{j=1}^{N} y_{ij} \le S_i x_i \qquad i = 1, \dots, N \\
& y_{ij} \ge 0 \qquad i = 1, \dots, N,\ j = 1, \dots, N \\
& x_i \in \{0, 1\} \qquad i = 1, \dots, N.
\end{aligned}$$

The first constraint "$\sum_{i=1}^{N} y_{ij} = d_j$" ensures that the demand in the i-th city is met. The constraint "$\sum_{j=1}^{N} y_{ij} \le S_i x_i$" ensures that the total number of pallets leaving city i is less than 0 if there is no DC opened in city i and less than S_i if a DC is opened. Nonnegativity of the transport amount is guaranteed by the constraints "$y_{ij} \ge 0$" and the binary nature (open or closed) of a DC by "$x_i \in \{0, 1\}$".